English for mathematical writing and presentations

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Contents

1: Style

1.1. Stylistic differences

Mathematics papers follow different style conventions to articles written in other disciplines. There are also various types of mathematics paper, which differ in style. Always check the style guide of your chosen journal.

1. Discuss the questions.

1. How would you describe the "mathematical" writing style?
2. Can you identify different varieties of mathematical prose?
3. How is mathematical writing different from other forms of writing? How is it similar?

2. Look at this general advice for writing mathematics papers. Can you explain the advice? Do you have any other general advice for writing mathematics papers?

General advice for mathematical writing

- Write for your audience
- Use paragraphs
- Use grammatically correct English
- Write clearly and simply
- Include motivation
- Avoid idioms
- Avoid most mathematical abbreviations
- Check your work

1.2. Formal or informal?

Using a formal style in your mathematical writing will ensure consistency of tone and register, especially if you are writing in teams.

3. Suggest more formal alternatives to these informal words.

1. a lot
2. big
3. it's
4. can't
5. thing
6. check
7. get better
8. go up
9. anyway
10. bad

However, you also shouldn't sound stuffy or verbose. Don't use "ergo," "thusly," "or other pompous words.

4. Suggest simpler alternatives to these words.

1. myriad
2. gargantuan
3. paucity
4. oeuvre
5. indubitably
6. aforementioned
7. henceforth
8. inter alia
9. infelicitous
10. elucidate

1.3. Active or passive?

A common tendency in technical and scientific writing is to overuse the passive voice.

Active voice sentence = actor + verb + target

- Wojtek kicked the ball

Passive voice sentence = target + verb + actor

- The ball was kicked by Wojtek

Active voice provides the following advantages:

- Active voice is generally shorter than passive voice.
- Active sentences are generally processed more easily.
- Some passive voice sentences omit an actor altogether, which forces the reader to guess the actor's identity.

There are times, however, when the passive voice is better for presenting an idea. For example:

- This problem has been studied by many researchers.
- In [2], the function f is approximated by linear functions.

Mentioning the problem or the equation number first highlights this important information.

5. Identify the actor (if present), verb, and complement in these sentences. Which sentences use the passive voice?

1. The nonlinearity R in Example 1.1 satisfies the hypotheses in Theorem 1.2.
2. Suppose that λ0 is a simple eigenvalue of L.
3. By a path of solutions in S is meant $\{\gamma(t) : t \in [0,1]\}$, where $\gamma : [0,1] \to S \subset \mathbb{R} \times X$ is continuous.
4. The methods introduced by Rabinowitz [**18**] lead easily to global bifurcation at eigenvalues of even multiplicity for certain nonlinearities [**20**].
5. It is easily seen that a bounded linear operator $L : X \to X$ has gradient structure if and only if…
6. Example C is a simplified version of Bohme's example [**5**] with added structure to ensure that all path-connected sets of non-trivial solutions are singletons.
7. The technique developed in the present paper may be adapted to elaborate on that observation.
8. Let $f : \mathbf{R} \to \mathbf{Z}$ be defined by $f(x) = \lceil 2x \rceil + \lceil 2x \rceil$.

6. Improve the unnecessary or excessive use of passive voice in these sentences.

1. In this paper is presented a simple example in which the hypotheses of the global bifurcation theorem are satisfied.
2. In the next example, it will be shown that the global continuum may not be path-connected away from the bifurcation point.
3. It is sufficient to show that in none of the cases is the conjecture disproved.

1.4. Personal pronouns

A common difficulty is deciding what to call yourself, as the author. The following points are often made:

- Mathematics papers are often written individually, but single authors very commonly use the plural pronoun "we".
- Most authors avoid using the word "I". The use of "one" as a pronoun can sound too formal and may be confusing.
- The use of "you" to refer to the reader or to people in general is considered informal.
- Pronouns can be avoided by using the passive. Always be clear and consistent in your use of personal pronouns.

7. Why is the plural pronoun "we" so common in mathematical writing, even in single-authored works?

8. Highlight the personal pronouns (I, me, you, we, etc.) in the abstract in Fig. 1 [29]. Discuss why the pronouns might be confusing or inconsistent.

9. Re-write the abstract to standardize the use of personal pronouns.

10. What other changes would be necessary to make the text more formal?

A MINUS SIGN THAT USED TO ANNOY ME BUT NOW I KNOW WHY IT IS THERE (TWO CONSTRUCTIONS OF THE JONES POLYNOMIAL)

Abstract. We consider two well-known constructions of link invariants. One uses skein theory: you resolve each crossing of the link as a linear combination of things that don't cross, until you eventually get a linear combination of links with no crossings, which you turn into a polynomial. The other uses quantum groups: you construct a functor from a topological category to some category of representations in such a way that (directed framed) links get sent to endomorphisms of the trivial representation, which are just rational functions. Certain instances of these two constructions give rise to essentially the same invariants, but when one carefully matches them there is a minus sign that seems out of place. We discuss exactly how the constructions match up in the case of the Jones polynomial, and where the minus sign comes from. On the quantum group side, one is led to use a non-standard ribbon element, which then allows one to consider a larger topological category.

Figure 1. Personal pronouns in an abstract [29].

1.5. Idioms

Idioms can be confusing, especially for international audiences, and are considered informal.

11. Match these idioms to their more universal alternatives.

1.	every which way	finally / lastly
2.	in the ballpark of	easy
3.	a piece of cake	a guaranteed outcome
4.	few and far between	briefly / to summarize
5.	needless to say	of course / obviously
6.	last but not least	■
7.	Bob's your uncle	haphazardly / in all directions
8.	to cut a long story short	well known
9.	old hat	rare / infrequent
10.	a dead cert	about / approximately

1.6. Abbreviations

Abbreviations are useful for writing on the blackboard and taking notes, but should usually be avoided in written mathematical exposition, especially in formal contexts.

12. Match these abbreviations to their long forms.

1.	iff	without loss of generality
2.	w.r.t.	with reference to
3.	w.l.o.g.	almost everywhere
4.	s.t.	the following are equivalent
5.	TFAE	almost all
6.	pf	no need to show
7.	NTS	if and only if
8.	i.e.	meaning that
9.	a.e.	such that
10.	a.a.	proof

13. Which of these abbreviations are commonly used in mathematical writing? Which should be avoided?

1.7. Structure [22]

There many possible structures for mathematics papers, depending on the field, the length of the paper, the journal requirements, etc. A basic structure is as follows:

1. Title and abstract
2. Introduction
3. Body
4. References

14. Why might some papers contain the following headings?

- Table of contents
- Foreword
- Final remarks
- Conclusions
- Acknowledgements
- Appendices

Longer papers may in addition be divided into sections. Sections are normally given numbers and titles. When referring to a numbered section, remember to capitalize the word "Section" (e.g. "see Section 4").

15. Look at some papers that won the David P. Robbins Prize, one of the MAA's Writing Awards. Notice how the papers are structured, their use of sections and subheadings, etc.

2. Words

2.1. Mathematical vocabulary

1. Work in groups. Take it in turns to explain one of the terms on the list, without using the term itself. Can your teammates guess the terms?

1. Axiom
2. Conjecture
3. Corollary
4. Function
5. Identity
6. Lemma
7. Proposition
8. Theorem
9. Hypothesis
10. Proof
11. Contradiction
12. Equivalence
13. Negation
14. Implication
15. Conclusion
16. Argument
17. Argumentation
18. Concept
19. Notion
20. Notation

2. Add keywords related to your research to the list. Can your classmates explain the terms?

2.2. Confusing words

Some words are confused in mathematical writing because they sound the same or are spelled similarly, leading to errors in transcription or interpretation. This can be particularly problematic in mathematics, where precision and accuracy are paramount.

3. Choose the best option to complete each sentence.

1. **Discrete/Discreet** data is non-continuous.
2. **Their/There** are a number of solutions to this problem.
3. **This/These** solutions are valid for a broad class of problems governed by equations of the type (1.1).
4. A curve **which/witch** is not of Type I is of Type II.
5. Now we can define the **complement/compliment** of a set.
6. If we **compare/compere** the exponents, we find that they are equal.
7. Let us start with a simple exponent and **its/it's** derivative.
8. Near the boundary, all the methods **loose/lose** the second order.
9. The first explicit formulation of the **principal/principle** of induction was given by Pascal in his *Traité du triangle arithmétique* (1665).
10. The notation $a > b$ means that a is greater **than/then** b.

4. Are there any other words you sometimes get confused?

2.3. Reference words

Reference words usually replace nouns, but can also refer back to whole clauses, sentences, and parts of text. Always ensure that the antecedent of any reference words is clear. If in doubt, repeat the noun or provide a synonym.

5. Choose the best option to complete each sentence.

1. The underlying space of X' is homeomorphic to **that/the one** of X
2. The exactness of G/Go implies **that/the exactness** of G.
3. We divide by the entropy of X or the entropy of Y, whichever is the larger of **the two/them.**
4. The results in the current paper do not follow from **those/the ones** in [**8**].
5. Suppose that you have two arrays of integers, A and B. **The former/*A*** has length n and **the former/*B*** has length m.
6. There are only eight possible scenarios. **These / They** are listed in Table 1.
7. We can use the Final Value Theorem to see how the error will respond in the long term. **This/It** is shown in Equation 2.
8. It was shown by W.T. Tutte that these necessary conditions are sufficient. The proof of **this result/this** is too difficult to be given here.

2.4. Quantifiers [19]

Quantifiers are adjectives or adjectival phrases that tell us how much or how many we have of something. Quantifiers are an essential part of mathematical writing as they allow us to make precise and concise statements about mathematical objects and their properties.

6. Choose the best option to complete each sentence.

1. **Every/All** prime number other than 2 is an odd number.
2. For **every/any** two rational numbers a and b, a + b is also a rational number.
3. In a linear equation, **each/every** term is either a constant or the product of a constant and a single variable.
4. Unfortunately, there are **a few/few** cases where this has been done successfully.
5. There **is/are** a large number of solutions to the 4 × 4 puzzle.
6. **Either/Or** x or y must be equal to 0.
7. **In either case we can/In neither case can we** get 3 as a sum.
8. The author suggests two options, **neither/none** of which is optimal.
9. There are **many/much** numbers that divide 109 with a remainder of 4.
10. No two members of the set **are/are not** congruent.

3. Sentences

3.1. Wrong word order [32]

In English, the subject almost always precedes the verb. The subject normally goes before the direct object. Try to keep the verb and the direct object together. Adjectives are usually placed before the nouns they modify. Past participles usually go after the noun.

1. Correct the mistakes.

1. The described above equation.
2. Its both solutions.
3. The two first rows.
4. Theorem 3 we shall prove in Section 1.
5. We can prove easily Theorem 1
6. We will prove in Section 3 Theorem 2.
7. Consider the two following statements.
8. Let f be such a function that f and f' are nondecreasing and convex.

3.2. Wordiness

Wordiness means using unnecessary words and longer phrases. You should always re-read and edit your text to remove redundancy.

2. Shorten these phrases to avoid wordiness.

1. Let us first observe that
2. We will first calculate
3. Hence it follows that
4. By virtue of (3)
5. Due to the fact that
6. By relation (3)
7. We can tentatively suggest
8. There are a large number of
9. What is more,
10. Despite the fact that

Sometimes, however, wordiness is welcome. In an opening sentence, for instance, it is perhaps better to write

- Let x be a point in a metric space X,

than

- Let $x \in X$, where X is a metric space

In this way, appropriate notation may be introduced.

3.3. Capital letters

A common mistake is to begin a sentence with a symbol or a variable. This can be confusing, especially if the previous sentence ends with a symbol. Readers expect the start of each sentence to be signaled by a capitalized word, so add text or restructure sentences as needed to begin each sentence with a word. Do not capitalize a symbol to indicate the beginning of a sentence.

3. Modify these sentences so they do not start with a small letter.

1. $2^n - 1$ is not prime for $n > 2$.
2. $x^2 - 1x - 90 = 0$ has solutions {a, b}.
3. f and m are parallel lines.

3.4. Commas

English uses commas less often than many other languages.

- Use a comma after all introductory elements.
- Any element before the main clause should be punctuated with a comma.
- Place commas around non-defining clauses (which may be removed without harming the sense).
- Do not put comma around clauses that define (provide essential information about) part of the sentence.
- Use commas to separate list items, including the Oxford comma to separate the last item in a list of three or more items, after 'and' or 'or'.

4. Decide if commas are needed before the letters in bold.

1. The random variable $\boldsymbol{x}$ is assumed to be distributed normally.
2. In [**2**] $\boldsymbol{x}$ is the dummy variable.
3. For all $x \in \mathbb{R}$ **a**nd for all $y \in \mathbb{R}$ $\boldsymbol{x} + y = 4$.
4. We shall now prove **t**hat 2 is measurable.
5. The fact **t**hat f has radial limits **w**as proved in [**4**].
6. This theorem **w**as proved in 1872 by Émile Borel **n**ot by Eduard Heine.
7. There is a polynomial $q(x)$ **s**uch that $p(x) = q(x^2)$.

8. Let us use this example **t**o understand the basic terminology associated with…
9. In math **a** variable is a letter **u**sed in place of an unknown number.
10. The most common examples of variables are x, y, and $\boldsymbol{z}$.

3.5. Short sentences

> In modern mathematical writing, short sentences are preferred. Focus each sentence on a single idea, thought, or concept.

5. Rewrite this translation from 1836 [7] of a classic text by Leonard Euler (1707–1783), written originally in Latin. Don't revise it too much, just split the text into a series of shorter sentences with other minor modifications where necessary.

Since all numbers which it is possible to conceive are either greater or less than 0, or are 0 itself, it is evident that we cannot rank the square root of a negative number amongst possible numbers, and we must therefore say that it is an impossible quantity. In this manner we are led to the idea of numbers which from their nature are impossible; and therefore they are usually called "imaginary quantities", because they exist merely in the imagination.

All such expressions as $\sqrt{-1}$, $\sqrt{-2}$, $\sqrt{-3}$, $\sqrt{-4}$, etc. are consequently impossible, or imaginary numbers, since they represent roots of negative quantities; and of such numbers we may truly assert, that they are neither nothing, nor greater than nothing, nor less than nothing; which necessarily constitutes them imaginary, or impossible.

But notwithstanding this, these numbers present themselves to the mind; they exist in our imagination, and we still have a sufficient idea of them; since we know that by $\sqrt{-4}$ is meant a number which, when multiplied by itself, produces -4; for this reason also, nothing prevents us making use of these imaginary numbers, and employing them in calculation.

3.6. Mathematical statements

> There is broad consensus that ordinary prose is much more effective at communicating with human readers than formal symbolic statements. Although you might initially construct your proof in the form of symbolic statements, when you write it up you should use complete sentences organized into paragraphs.

6. Write each statement in plain English. Do not use any symbols except the letters that denote elements of the universe.

1. $\forall x \in \mathbb{R}\ \forall y \in \mathbb{R}(x\ 6 \neq y) \Rightarrow (x + y\ 6 = 0)$, where the universe is the real numbers.
2. $\exists s, \forall t, p(s) \wedge [(t\ 6 = s) \rightarrow \neg p(t)]$, where the universe of s and t is the collection of all students who completed this course last fall, and $p(s)$ is the assertion "s got 100% on the final exam".

4. Paragraphs

4.1. Characteristics of effective paragraphs

1. Work in groups. Answer the questions.

1. What is the purpose of a paragraph?
2. What textual features do you think are necessary for developing a coherent paragraph?
3. What kind of content can a paragraph contain?

4.2. Paragraph breaks [21]

Paragraph breaks signal places to stop and think, and show that a new thought has begun.

2. Insert paragraph breaks in the following text [23]. Mark the place in the text where the paragraph break should be with a pilcrow ¶ in the margin.

Paterson and Zwick [**14**] describe a family of balanced *n*-block stacks that achieve an overhang of about $(3n/16)^{1/3}$ $0.57n^{1/3}$. More precisely, they construct for every integer $d \geq 1$ *a* balanced stack containing $d(d - 1)(2d - 1)/3 + 1$ $2d^3/3$ blocks that achieves an overhang of $d/2$. Their construction, for $d = 6$, is illustrated in Figure 7. The construction is an example of what they term a brick-wall stack, which resembles the simple "stretcher-bond" pattern in real-life bricklaying. In each row the blocks are contiguous, with each block centered over the ends of blocks in the row beneath. Overall the stack is symmetric and has a roughly parabolic shape, with vertical axis at the table edge. The stacks of [**14**] are constructed in the following simple manner. A *t-row* is a row of t adjacent blocks, symmetrically placed with respect to $x = 0$. An *r-slab* has height 2r − 3 and consists of alternating *r*-rows and $(r - 1)$-rows, the bottom and top rows being *r*-rows. An *r*-slab therefore contains $r(r - 1) + (r - 1)(r - 2) = 2(r - 1)2$ blocks. A 1-*stack* is a single block balanced at the edge of the table; a *d-stack* is defined recursively as the result of adding a *d*-slab symmetrically onto the top of a $(d - 1)$-stack. The construction itself is just a *d*-stack and so has overhang $d/2$; its total number of blocks is given by $n = 1 + 2(r - 1)2 = d(d - 1)(2d - 1)/3 + 1$. It is shown in [**14**], using an inductive argument, that *d*-stacks are balanced for any $d \geq 1$. Why should a parabolic shape be appropriate? Some support for this comes from considering the effect of a block in spreading a single force of f acting from below into two forces of almost $f/2$ exerted upwards from its edges. This spreading behavior is analogous to a symmetric random walk on a line or to difference equations for the "heat-diffusion" process in a linear strip. In both cases we see that time of about d^2 is needed for effective spreading to width *d*, corresponding to a parabolic stack profile.

4.3. Topic sentences

Topic sentences are one way to indicate the main point of a paragraph. The topic sentence is usually the first sentence in the paragraph.

3. Identify the topic sentences in the text about balanced stacks in Exercise 2.

4.4. Paragraph building

4. Sentences A–E are not in the correct order. Rearrange the sentences to form a paragraph, with the first sentence being the topic sentence.

A By taking these courses, mathematicians can learn how to communicate complex ideas clearly and effectively, which can help them advance their careers and make meaningful contributions to the field.

B Communication skills are crucial for mathematicians to effectively convey their ideas, theories, and proofs to a wider audience.

C These courses generally focus on the key skills for mathematicians of writing and making presentations.

D They may also include discussions and feedback sessions to help students improve their communication abilities.

E Fortunately, there are several courses available that focus on teaching communication skills to mathematicians.

5. Which words and phrases helped you to identify the topic sentence? How did you decide the correct order of the other sentences?

6. Do you think that communicating about mathematics also helps you learn mathematics? Which kinds of tasks do you think work best?

7. Put the sentences from the paragraph below (A–E) into the correct order. Then compare the points you made in Exercise 6 to the points made in the paragraph.

A Overall, using communicative activities in mathematics teaching can be an effective strategy for promoting student learning and engagement.

B Research has shown that incorporating communicative activities into mathematics instruction can have a positive impact on student learning outcomes.

C This approach can lead to deeper understanding of mathematical concepts, as students are encouraged to articulate their reasoning and engage with their peers' ideas.

D These activities involve promoting discussion and collaboration among students, allowing them to share their understanding and problem-solving strategies with each other.

E Additionally, communicative activities can increase student motivation and engagement in mathematics, as they provide opportunities for students to apply their learning in real-world contexts and make meaningful connections to their lives.

4.5. Paragraph writing

Find a research article related to your research interests. Write two or three paragraphs presenting the main idea of the article.

5. Titles and abstracts

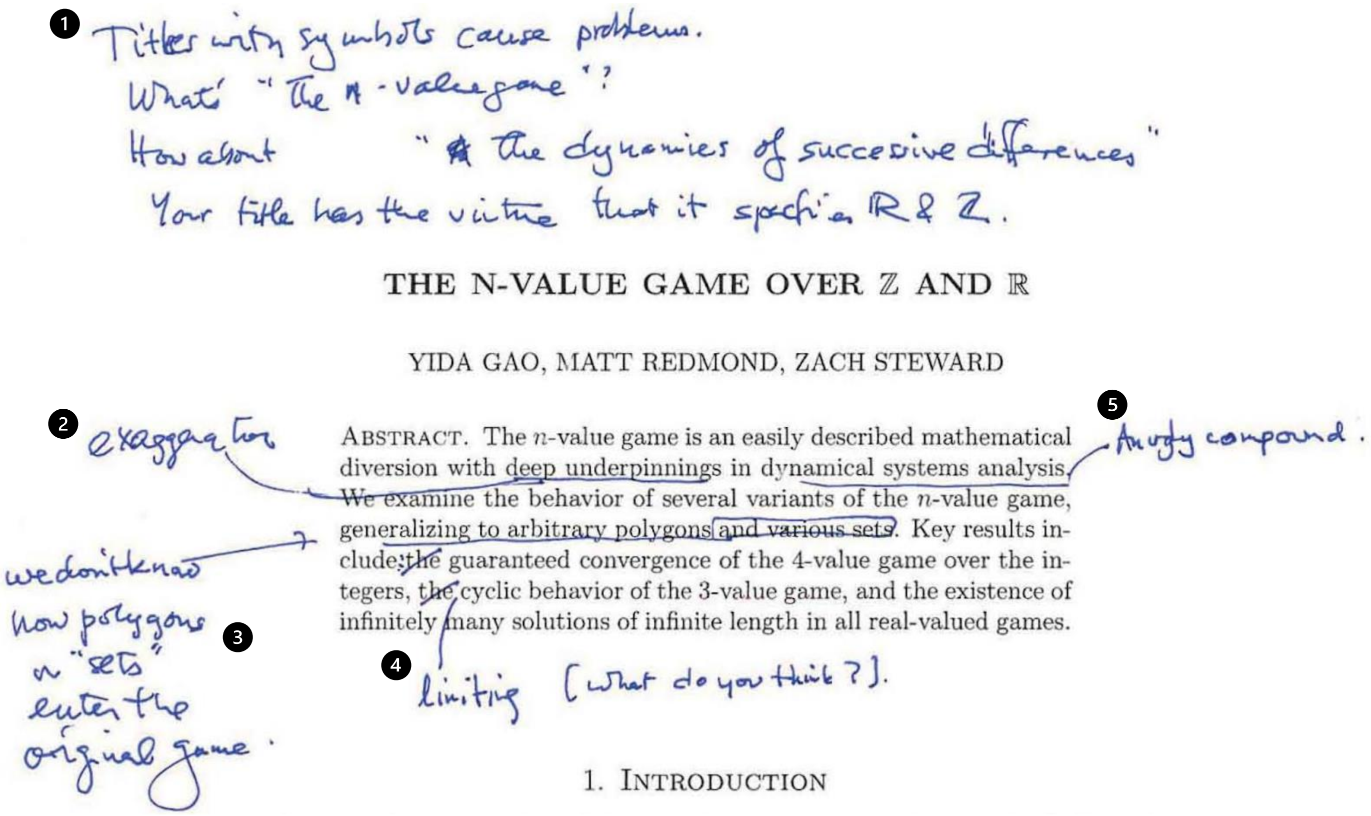
5.1. About titles and abstracts

1. Why are titles and abstracts important?

2. Titles and abstracts are often the last parts of the article authors write. Why?

5.2. Revising titles and abstracts

3. Imagine you have received this feedback on your paper. The feedback was given by Prof. Haynes Miller to MIT students on a mathematical writing course [23]. Can you explain the comments?


THE N-VALUE GAME OVER $\mathbb{Z}$ AND $\mathbb{R}$

YIDA GAO, MATT REDMOND, ZACH STEWARD

ABSTRACT. The n-value game is an easily described mathematical diversion with deep underpinnings in dynamical systems analysis. We examine the behavior of several variants of the n-value game, generalizing to arbitrary polygons and various sets. Key results include: the guaranteed convergence of the 4-value game over the integers, the cyclic behavior of the 3-value game, and the existence of infinitely many solutions of infinite length in all real-valued games.

1. INTRODUCTION

①	Titles with symbols cause problems. What's "the n-value game? How about "the dynamics of successive differences" your title has the virtue that it specifies ℝ and ℤ
②	Exaggeration
③	We don't know how polygons or "sets" enter the original game.
④	Limiting (what do you think?).
⑤	An ugly compound.

4. Now read the revised version of the title and abstract. What changes have been made? Can you explain the changes? ‘

THE DYNAMICS OF SUCCESSIVE DIFFERENCES OVER $\mathbb{Z}$ AND $\mathbb{R}$

YIDA GAO, MATT REDMOND, ZACH STEWARD

ABSTRACT. The n-value game is a dynamical system defined by a method of iterated differences. In this paper, we examine the behavior of several variants of the n-value game, and prove a few key results: the 4-value game over the positive integers is guaranteed to converge to a fixed point; the 3-value game over the positive integers is guaranteed to exhibit cyclic behavior; for all n, there exist infinitely many non-cycling n-value games over the positive reals with infinite length.

5. What is good about the title and abstract below? What could be improved?

ENUMERATION OF COLORINGS OF ARCS OF PROJECTIONS OF A KNOT K

Al Dough, Bea Row, and Cee Low

ABSTRACT. We look at possible ways of coloring the arcs in the plane projection of a knot. We prove that this gives rise to a knot invariant, which can distinguish infinitely many different equivalence classes of knots.

6. Watch this video (8:00–17:10) and compare your ideas to the discussion with MIT students [21].

5.3. Compound nouns

A compound noun is a noun modified by an adjective, verb, or another noun. Compound nouns can be one word, like *database*, or two words, like *computer science*. If two or more words are used to modify a noun (or a compound noun) they may be joined with a hyphen, as in *billion-dollar particle collider*.

7. Re-write the following as compound nouns.
1. The set of all factors of a given number.
2. A reference for a position on a grid.
3. A language for programming a computer.
4. A set of data.
5. A teacher of mathematics.
6. The sign used to indicate mathematical division.
7. A table that shows you the results of multiplying numbers.
8. A problem with two parts.
9. An article published in a journal.
10. Classifications of subjects in the journal *AMS.*

8. Shorten these the noun strings by replacing the prepositions and articles, using verb forms, etc.
1. A logarithmic function integral list
2. An angular momentum conservation equation
3. A continuous-time dynamical system phase portrait.

5.4. Articles

There are three types of articles in English:

Definite article (the): used for things we know about or which are unique.

Indefinite article (a/an): used for things that are introduced for the first time or not specific. *An* is used before words that start with a vowel sound.

Zero article (–): used for unspecified uncountable nouns, plural nouns, abstract nouns, and proper noun.

9. Complete these sentences with a / an / the / – .
1. We conclude this section with ___ useful lemma.
2. Three blocks, for example, can easily be used to obtain ___ overhang of 1.
3. Theorem 2 has ___ very important converse, _____ Radon-Nikodym theorem.
4. Our present assumption implies that ___ last inequality in (8) must actually be _____ equality.
5. ___ only additional feature is ___ appearance of a factor of 2.
6. ___ earth has an average density 5.5 times that of ___ water.
7. ___ r-slab therefore contains $r(r-1)+(r-1)(r-2) = 2(r-1)^2$ blocks.
8. ___ proof of ___ Lemma 4.4 is given in ___ Section 5.4.

5.5. Writing titles and abstracts

10. Write a title and abstract for a research project you are working on.

> The length of an abstract should reflect the length of the paper: the number of lines should be about 0.5 times the number of pages [25].

Useful phrases for titles

- On...
- Short proof that...
- New examples of...
- A remark on...
- A survey on...
- A short survey on...

Useful phrases for abstracts

- We are interested in finding…/ wish to investigate
- We propose / study
- For… what is…?
- … is…

6. Introductions

6.1. Characteristics of effective introductions

1. Decide which of the following are characteristics of a good introduction.

- ☐ Indicates the article’s main result(s)
- ☐ Indicates why the results are important
- ☐ Previews the structure
- ☐ Gives technical definitions
- ☐ Provides figures illustrating some special cases
- ☐ Gives detailed examples
- ☐ Contains proofs

6.2. Opening sentences [4, 25]

> Start by setting up the problem and statements of the main results. Define unfamiliar terms and give a few technical definitions if they are needed to understand the results. If you are resolving a conjecture or question, state it (with attribution).

2. Read these opening sentences adapted from papers that won the David P. Robbins Prize, one of the MAA's Writing Awards.

A. An alternating sign matrix (ASM) is a square matrix in which each entry is 0, 1, or −1, and along every row and column the nonzero entries alternate in sign and have a sum of 1.

B. The most famous result of extremal combinatorics is probably the celebrated theorem of Turán [20] determining the maximal number $\mathrm{ex}(n; K_r)$ of edges in a K_r-free graph on n vertices.

C. We present a combinatorial solution to the *Carpenter’s Rule Problem*: how to plan non-colliding reconfigurations of a planar robot arm.

D. The starting point of this note is a result proved in [1], which states that the cotangent manifold of a special symplectic manifold (M, J, ∇, ω) inherits, under some additional conditions, a hyper-Kähler structure.

E. In the historic conference *Combinatoire Enumerative* that took place at the end of May 1985 in Montréal, Richard Stanley raised some intriguing problems about the enumeration of plane partitions (see below), which he later expanded into a fascinating article [11].

3. Which of the sentences (A–E) contain the following?

1. A technical definition
2. A famous result
3. Historical perspective
4. An open research problem
5. The motivation of the study

4. Which of the opening lines A–E would make you want to read the rest of the article? Why?

5. Write the opening few sentences for your article.

> Mathematical writing tends to be so plain and simple, this is your only space to write something more elaborate. Feel free to discuss the big picture, make links to other fields of study, or refer to general physical or philosophical principles underlying your work, etc. [4].

6.3. Literature review

6. What are the functions of a literature review? Tick the boxes that apply.

- ☐ Indicate how a paper’s results further research within the field.
- ☐ Identify a gap in current knowledge and understanding that your research helps to fill.
- ☐ Give readers confidence that the authors are familiar with the relevant literature.
- ☐ Help readers new to the field (e.g., graduate students).
- ☐ Give credit to authors who influenced your work.
- ☐ Indicate to readers where they can find out more about the topic.
- ☐ Introduce concepts that will be needed to understand the statement or the significance of the paper's main result(s).

Useful phrases for literature reviews

- Author [**1**] was the first to consider…
- Author [**2**] asked about… and proved…
- Last year, Author [**3**] showed that…
- Several analyses of… have been proposed [**4,5,6**].
- We should also mention a series of papers [**12,13,14**] on…
- It can be deduced from [**7**] that…

- In [**8**], Author gave a proof of…
- The present article is strongly influenced by [**9**].
- This article can be seen as a natural prolongation of [**10**].
- This paper builds on the tools in [**11**].
- The standard work on… is [**15**]
- For a deeper discussion of… we refer the reader to [**16**].

7. Write a paragraph reviewing the literature for your article.

6.4. References [21]

Plagiarism means copying phrases or sentences from a source without attribution. It is also inadmissible to reproduce an argument, even in your own words, without some form of attribution. It is common to refer to secondary literature or textbooks as sources for arguments. Anything not referenced or explicitly labeled as known is assumed to be original.

8. Which citation styles are the most commonly used in mathematical writing? Which system is used by the journal you are writing your next paper for?

9. Imagine you are a reviewer. What comments would you make about the reference list below?

REFERENCES

[1] Wikipedia contributors, "Descartes' rule of signs," Wikipedia, The Free Encyclopedia, https://en.wikipedia.org/wiki/Ren%C3%A9_Descartes

[2] Weisstein, Eric W. "Polynomial Roots." From MathWorld – A Wolfram Web Resource. https://mathworld.wolfram.com/PolynomialRoots.html

LaTeX supports several standard citation systems. The easiest uses the command "`\begin{thebibliography}{99}`" where you want the bibiliography. Each bibliograhic item is created by a command like

```
\bibitem{fenn-rourke}
R.~Fenn and C.~Rourke,
Racks and links in codimension two,
J. ~of Knot Theory and its Ramifications
1 (1992) 343--406.
```

The bibliography is ended with "`\end{thebibliography}.`" The item is referenced in the text by "`\cite{fenn-rourke}.`" You need to execute LaTeX twice on the document in order to link the citation to the bibliographic item.

6.5. Purpose and main results

After reviewing the literature, the introduction then typically states the purpose of the paper: how will you address the gap or problem identified by the literature review? The purpose statement often highlights the novelty or contribution of the paper. It can also limit the scope of the paper, by saying what it will *not* cover.

10. Read these extracts from papers that won the David P. Robbins Prize [2, 5, 26, 28]. Which paper do you think is the most novel? Which is the least novel?

A. The proofs given in this paper of enumeration results for odd-order DASASMs share several features with known proofs of enumeration formulae for other symmetry classes of ASMs, such as those of Kuperberg [**31, 32**], Okada [**37**], and Razumov and Stroganov [39, 40]. However, the proofs of this paper also contain various new and **distinguishing characteristics**, which will be outlined in Section 1.3.

B. In summary, the only existing proofs for Problems (A) and (B) used heavy computer calculations in a **crucial** way. In this article we obtain a new explicit expression for *Q(x, y; z)*, from which we derive the first "human proofs" of (A) and (B).

C. In this paper we enhance this latter proof with one more inductive argument and establish Conjecture 1 for $r = 3$ and *arbitrary* values of *t*. Like the "base" proof, this extension was worked out entirely in the framework of flag algebras developed in [19], and it is presented here within that framework. Our arguments in favor of this approach were carefully **laid out** in [**19**].

D. We strengthen and provide an algorithmic extension of the above mentioned Carpenter's Rule result of [**30**]. We show how to compute a path in configuration space, consisting in at most $\mathcal{O}(n^3)$ simple motions along algebraic curve segments, between any two polygon configurations. **Along the way,** we obtain a result of independent interest in Rigidity Theory. Namely, we characterize a family of planar infinitesimally rigid, self-stress-free frameworks called pointed pseudo-triangulations, which yield one-degree-of-freedom (1dof) *expansive* mechanisms when a convex hull edge is removed.

11. Match the words in bold to their synonyms 1–4.

1. described
2. important
3. unique features
4. in the process

12. Write a few sentences explaining the purpose and main results of your article. Consider these questions [25]:

- Does your result strengthen a previous result by giving a more precise characterization of something?
- Have you proved a stronger result of an old theorem by weakening the hypotheses or by strengthening the conclusions?
- Have you proven the equivalence of two definitions?
- Is it a classification theorem of structures which were previously defined but not understood?
- Does it connect two previously unrelated aspects of mathematics?
- Does it apply a new method to an old problem?
- Does it provide a new proof for an old theorem?
- Is it a special case of a larger question?

Useful phrases for purpose and main results

- Our purpose is to…
- It is not our purpose to…
- The main objective/aim of this work is…
- No attempt is made here to…
- In this paper, we prove…

- There are two main purposes of this paper. The first purpose is to...
- Here, we present a proof of…
- We show here that…
- It is also shown that...
- In particular, we focus on…
- More specifically, we show that…
- In this paper, we refine the tools in [9] to show that…
- We also analyze the examples in [10] and show that…

6.6. Roadmap

The introduction often concludes with a "roadmap" or brief outline of the paper. Because the accepted structure for research papers is much less rigidly defined for mathematics than for many other scientific fields, the roadmap helps the reader to navigate the paper and find information of interest.

13. Read this excerpt from another paper that won the David P. Robbins Prize, in 2011 [26]. Complete the gaps (1–7) with the phrases (A–G).

[1]____ In the next section, we present a precise mathematical definition of the overhang problem, [2]____ when a stack of blocks is said to be *balanced* (and when it is said to be stable). [3]____ we briefly review the Paterson-Zwick construction of stacks that achieve an overhang of order $n^{1/3}$. In Section 4 [4]____ a class of abstract "mass movement" problems and [5]____ these problems and the overhang problem. [6]____ we obtain bounds for mass movement problems that imply the order $n^{1/3}$ upper bound on overhang. We end in Section 6 with [7]____ and open problems.

A. In Section 3
B. we introduce
C. explaining in particular
D. The rest of this paper is organized as follows.
E. In Section 5
F. explain the connection between
G. some concluding remarks

14. Which tense is used to introduce the content of each section?

15. Write a roadmap for your article, briefly outlining the content of each section.

Useful phrases for roadmaps

- In what follows of the introduction, we…
- In the next section, we…
- Section 2 presents some preliminaries.
- Section 2 is intended to motivate our investigation of…
- Section 3 discusses the case…
- Section 4 provides a detailed exposition of…
- Section 4 expands on…
- Section 4 provides more details on…
- Section 4 explains the connection between…

7. Definitions

7. Formal and informal definitions

Definitions may be presented informally within the text or they may be set formally. Setting a definition formally makes the definition easier for readers to find if they need to refer back to it. Emphasize the individual words being defined using italics. The complete definition should not be italicized, because this would require emphasizing large parts of text [31].

1. Emphasize the words defined informally in this text using a straight line for *italics* [23].

We say that block B_i rests on block B_j if $B_i \cap Bj \neq \emptyset$ and $y_i = y_j + h$. If $B_i \cap B_j \neq \emptyset$ and $y_i = 0$, then B_i rests on the table. If B_i rests on B_j , we let $Ii\ j = Bi \cap Bj = [a_{i\,j}, b_{i\,j}] \times \{yi\}$ be their contact interval. If $j \geq 1$, then $a_{ij} = \max\{x_i, x_j\}$ and $b_{ij} = \min\{x_i + 1, x_j + 1\}$. If $j = 0$ then $a_{i0} = x_i$ and $b_{i0} = \min\{x_i + 1, 0\}$. The overhang of a stack is defined to be $\max_{i=1}^{n}(x_i{=}1(x_i + 1)$.

2. Write the term being defined formally in this text on the line in brackets, and underline the term in the text for *italics.*

Definition 2.1 (________________). Let B be a homogenous block of unit length and unit weight, and let a be the x-coordinate of its left edge. Let $(x_1, f_1), (x_2, f_2), \ldots, (x_k, f_k)$ be the positions and the magnitudes of the upward forces applied to B along its bottom edge, and let $(x'_1, f'_1), (x'_2, f'_2), \ldots, (x'_k, f'_k)$ be the positions and magnitudes of the upward forces applied by B along its top edge, on other blocks of the stack. Then B is said to be in equilibrium under these collections of forces if and only if

$$\sum_{i=1}^{k} f_i = 1 + \sum_{i=1}^{k'} f'_i\,, \quad \sum_{i=1}^{k} x_i f_i = \left(a + \frac{1}{2}\right) + \sum_{i=1}^{k'} x'_i f'_i\,.$$

The first equation says that the net force applied to B is zero while the second says that the net moment is zero.

A common mistake is to begin a paper by simply listing, without context, all definitions that will be needed in the paper. While such a section may act as a useful reference in a longer work, in most cases it's more helpful to readers if new terms are introduced in context and with some motivation.

3. Choose the best option.

1. a) A function f **is said to be** *continuous* if…
   b) **It is said that a function f is** *continuous* if…
2. a) We **say** that a function f is *continuous* if…
   b) We **state** that a function f is *continuous* if…
3. a) By a *block* we understand a set **having** the following properties.
   b) By a *block* we understand a set **with** the following properties.
4. a) A *bipartite graph***,** is a graph that is 2-colorable.
   b) A *bipartite graph* is a graph that is 2-colorable.
5. a) The *trace* of a square matrix **is defined to be** the sum of its diagonal entries.
   b) The *trace* of a square matrix **is defined by** the sum of its diagonal entries.

Useful phrases for definitions

- A set S is *dense* if...
- A set S is called *dense* if...
- We call a set *dense* if...
- The function f is defined by...
- We define T as…
- The length of a sequence is, by definition, ….
- By the length of T we mean...
- Let $E = Lf$, where f is...
- In this way we obtain what we shall call...

8. Proofs

8.1. Considerations for writing proofs [17, 18]

- **Label your theorems.** The usual convention is to set the word "Theorem" (etc.) in bold, with the statement of the theorem italicized.
- **State what you're proving.** Explain precisely, in one or more sentences, of what you are to prove and your assumptions (this preamble is sometimes very short or omitted if the very statement of the theorem allows).
- **Show where your proofs begin and end.** Each proof should begin with the word "Proof" and end with a distinctive symbol such as a box. In LaTeX there is a special environment: \begin{proof}...\end{proof}.
- **Include more than just the logic.** If you have to write a long sequence of formulas, break up the formulas with words and phrases explaining why one step follows from another, what you're doing and why. It is important to use a variety of different phrases and synonyms to present the proof elegantly and correctly.

Tip: use mathematical notation to convey facts and English to convey structure.

8.2. Examples of written proofs [19]

Proposition 8.2.1 *If the integer n is even, then n^2 is even.*

Proof. Suppose that the integer n is even. We shall prove that n^2 is even. It follows that there exists an integer k so that $n = 2k$. Then, $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$. Since $2k^2$ is an integer, n^2 is even. ∎

This direct proof begins by stating the goal and assumptions of the proof. The logical connections between the statements are marked by short words and phrases (*it follows, then, since*). A solid box is used to indicate the end of the argument.

Proposition 8.2.2 *If the integer n^2 is even, then n is even.*

Proof. We will prove the contrapositive that if n is not even, then n^2 is not even. Suppose that the integer n is not even—that is, it is odd. We want to show that n^2 is odd. Since n is odd, there exists an integer k so that $n = 2k + 1$. Then, $n^2 = (2k + 1)2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$. Since $2k^2 + 2k$ is an integer, n^2 is odd. ∎

This proof uses the *contrapositive method*. The writer starts off by saying that they are going to prove the contrapositive to the statement. *Do not skip this step!* It communicates to the reader that they should not expect you to prove the original statement directly. The author then writes out the contrapositive to the statement. Doing so, they initially use the expression "not even," which is common when presenting the contrapositive. Then, before switching, for clarity the they remind us that this means "odd." Because the logical equivalence of the final statement and the original proposition is obvious, there is no need for further details. This method is sometimes called giving an *indirect proof*.

Proposition 8.2.3 $\sqrt{2}$ *is not rational.*

Proof. Assume for the sake of contradiction that $\sqrt{2}$ is rational. Then there exist integers a and b so that $\sqrt{2} = a/b$. The integers a and b can be chosen so that the fraction a/b is in lowest terms, so that a and b have no common factor other than 1. In particular, a and b are not both even.

Since $\sqrt{2} = a = b$, we have that $2 = (a/b)^2 = a^2 = b^2$. By algebra, $2b^2 = a^2$. Therefore a^2 is even. By Proposition 1.4.2, a is even. Thus there exists an integer k so that $a = 2k$. It now follows that $2b^2 = a^2 = (2k)^2 = 4k^2$, so that $b^2 = 2k^2$. Therefore b^2 is even. By Proposition 1.4.2, b is even.

We have now derived the contradiction (a and b are not both even) and (a and b are both even). Therefore, $\sqrt{2}$ is not rational. ∎

This slightly longer *proof by contradiction* is split into three short paragraphs. One of the most important parts of a proof by contradiction is the very first part, where you should write out the negation of what you want to prove. This makes the proof easier to read and reduces the chance that you misunderstand the original statement. The second paragraph explains why the new assumptions are false. Part of the justification is provided by reference to a previous proof, which is cited by number (Proposition 1.4.2). The final paragraph summarizes the contradiction and presents the conclusion (i.e., the original proposition is true).

Proposition 2.4.4 *If the integer n^2 is a multiple of 3, then n is a multiple of 3.*

Proof. We prove the contrapositive: if n is not a multiple of 3, then n^2 is not a multiple of 3. Suppose n is not a multiple of 3. Then the remainder when n is divided by 3 equals 1 or 2. This leads to two cases:

Case 1. The remainder on dividing n by 3 equals 1.

Then, there exists an integer k so that n = 3k+1. Hence n2 = (3k+1)2 = 9k2 + 6k + 1 = 3(3k2 + 2k) + 1. Since $(3k^2 + 2k)$ is an integer, the remainder on dividing n^2 by 3 equals 1. Therefore n^2 is not a multiple of 3.

Case 2. The remainder on dividing n by 3 equals 2.

Then, there exists an integer k so that n = 3k+2. Hence n^2 = (3k + 2)2 = 9k2 + 12k + 4 = 3(3k2 + 4k + 1) + 1. Since $(3k^2 + 4k + 1)$ is an integer, the remainder on dividing n^2 by 3 equals 1. Therefore n^2 is not a multiple of 3.

Both cases have now been considered. In each of them, we have shown that n^2 is not a multiple of 3. It now follows that if n is not a multiple of 3, then n^2 is not a multiple of 3. This completes the proof. ■

This *proof by cases* uses all the other writing guidelines. In addition, new paragraphs with labels (*Case 1, Case 2*) make it clear where each case begins. This can also be done by starting a paragraph with a phrase such as "In the case where…"

8.3. Writing proofs [19]

Prove that:

1. The sum of two even inters is even.
2. The sum of an even integer and an odd integer is odd.
3. The sum of two odd integers is even.
4. The product of two even integers is even. Further, it is a multiple of 4.
5. The product of an even integer and an odd integer is even.
6. The product of two odd integers is odd.
7. If a and b are integers such that a + b is even, then a and b are both even or both odd.
8. If a and b are integers such that $a + b$ is odd, then a is even and b is odd, or a is odd and b is even.
9. If a and b are integers such that ab is even, then a is even or b is even.
10. If a and b are integers such that ab is odd, then a and b are both odd.

Useful phrases for writing proofs

- First, prove / show / recall that...
- To prove this, let...
- To this end, consider... / To do this, take / For this purpose, we set...
- To deduce (3) from (2), take...
- Our proof starts with the observation that...
- The procedure is to find...
- The proof is straightforward / quite involved.
- The proof is left to the reader.
- The main idea of the proof is to take...
- Suppose, contrary to our claim, that...
- On the contrary, …
- Conversely, …
- Assume the formula holds for...
- We give the proof only for the case...
- We give only the main ideas of the proof.
- We will prove this result by proving the contrapositive of the statement.
- We will prove this statement using a proof by contradiction.
- We will use a proof by contradiction.
- By contradiction; assume that…
- We will prove the contrapositive of this statement, namely, that…
- By contrapositive; we will instead prove that…
- Assume for the sake of contradiction that…
- By a similar argument, ...
- It follows that...
- Hence, ...
- We thus get...
- We conclude from (5) that...
- In the same manner we can see that...
- The rest of the proof runs as before
- Consider... Define... Choose... Let... Fix... Set...
- Subtracting (3) from (5) yields...
- Combining (4) and (5) we obtain...
- Applying (5), we see that...
- We next show/prove that...
- Our next goal is to...
- The task is now to find...
- It suffices to show that...
- We need only consider...
- It remains to prove that...
- ..., which is impossible.
- ..., which proves the theorem.
- ..., which completes the proof.
- ..., which establishes the formula.
- ..., and the proof is complete.
- ..., and (3) is proved.

9. Figures and tables

9.1. Formatting figures and tables [21]

Although figures can take a long time to create, a well-constructed figure can be quite helpful to readers. If you have a picture in mind as you write, consider including that figure in the paper.

1. Read the guidelines for drawing figures and tables. Discuss anything
1. you didn't know before
2. you knew before
3. you think is interesting
4. you think is important
5. you disagree with

2. Discuss Fig. 2. What should be improved in terms of its format?

3. Discuss figure and tables you have created, or from and article related to your research interests.

9.2. Introducing figures and tables

4. Read the guidelines for introducing figures and tables.

5. Find where the tables and figures are mentioned in an article related to your research interests. Does the article follow the same guidelines?

6. Write a few sentences for a research article introducing and highlighting the main results given in Figure 2.

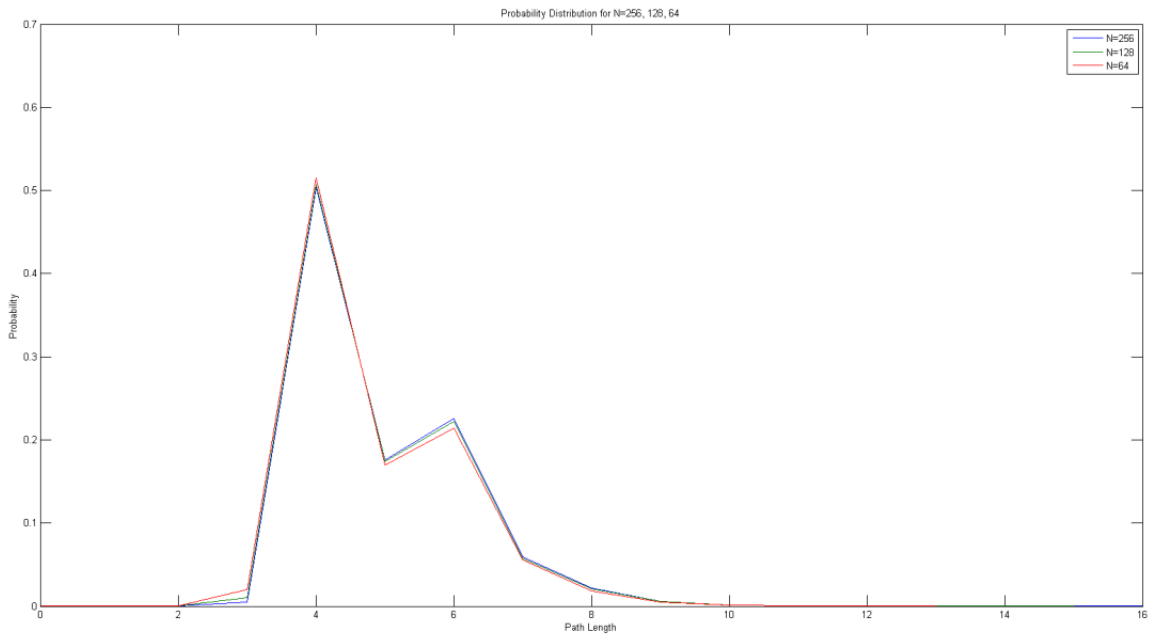
Useful phrases for figures and tables

- Table 1 displays…
- Figure 1 shows…
- … is shown in Figure 2.
- The data are given in Table 2
- Figure 1 should provide some intuition about…
- In Fig.3, …

Guidelines for formatting figures and tables

- Make sure the font size is sufficiently large.
- Label the axes in graphs.
- Make sure all author-defined acronyms, abbreviations, and symbols are defined both in the text and in the figure or table.
- Use different line colors or thicknesses (rather than different line styles, such as dashed or dotted lines) to make it easy to distinguish between lines.
- Be aware that colors may look different in different media, such as on a screen or when printed and photocopied. If you are using different shades of grey to distinguish between elements in a graph, make sure they are sufficiently different.
- Discrete data should be plotted using distinctive marks such as circles, boxes, or triangles
- Remove all unnecessary elements and ornamentation.
- Do not put boxes/frames around figures or tables.
- If you copy a picture from elsewhere, cite the source in the figure's caption. If you modify a picture to fit the context of your paper, the citation should say "modified from [Ref]."
- Figures and tables are normally positioned after the paragraph in which they are introduced in the text.
- Do not start a new section with a table or figure, without any introductory text.
- Figure captions are typically written below the figure (figure titles go above in a presentation).
- Table captions are typically written above the table.
- Write "Table" or "Figure" (with a first capital letter or fully capitalized) followed by the number corresponding to the order in which the item appears. Word and LaTeX have good features for automatic numbering.
- Most journals put a stop or a colon after the number of the figure or table, and begin the next sentence with a capital letter—e.g. "Figure 1. Distribution of path length for *n* = {256, 128, 64}".
- Omit any article at the start of the first sentence, e.g. "~~The~~ Distribution of path length for…"
- Make your captions as concise as possible.
- You are not always required by the journal to end the caption with a stop, but you must be consistent.

FIGURE 2. Distribution of Path Length for $n = \{256, 128, 64\}$



Guidelines for introducing figures and tables.

- Each figure should be mentioned within the text by number. Figures often move to different pages as you edit, so using a number is safer (and more conventional) than writing “above” or “below”.
- Make sure the purpose of including each table or figure is explained.
- When the words “table” or “figure” are followed by a number, the first letter is always capitalized (e.g. Table 1).
- Do not put a stop after the number of a figure or a table in the text (i.e. do not write “Figure 1. shows…”).
- Do not abbreviate the word “figure” if it is not followed by a number.
- Be consistent if you are abbreviating Figure to Fig. The plural abbreviated form can be “Fig.” or “Figs.”
- The abbreviation “Tab.” for “Table” is discouraged in many style guides.
- Do not start sentences with the abbreviated form “Fig”.
- You can refer to figures and tables within the sentence, or in brackets like this: “The results showed a positive correlation (Fig. 2)” or “(see Fig. 2)”.
- Do not write “As it is shown in Figure 1…” or “As Figure 1 shows…”. Do write “As shown in Figure 1…” or “Figure 1 shows…”
- In your discussion of a table or figure that has been introduced already by its number, you may subsequently use the definite article “the” to refer to “the table” or “the figure” that has been mentioned before.

10. Presentations

10.1. Characteristics of effective presentations

1. Read one of these texts, then summarize the main points for the class.

- How to give a good 20 minute math talk
- Talks are not the same as papers
- So You Want to Give a Math Talk...
- Tips on giving talks
- How does one give a mathematical talk?

1. What were the most important points?
2. Was there anything that surprised you?
3. Was there anything you disagree with?
4. What was the best advice?
5. What will you do differently after reading the text?

10.2. Know your audience

> Whereas the audience for a research paper can be assumed to be familiar with the field, your audience for a presentation may include mathematicians from a variety of areas, who require more background and context.

1. Write 3 questions for the audience of your next presentation:

1. One question you think they will definitely know the answer to.
2. One question you think they might know the answer to.
3. One question you would be surprised they knew the answer to.

2. Find out what your audience knows about the topic of your next presentation.

10.3. Presentation introductions

> The beginning is a key moment in a talk, when you should have the audience's full attention. The kind of introduction you give will depend on the type of presentation, your audience, and the topic.

3. Decide why a presenter might decide to (or not to) do these things in the introduction to a research presentation.

- Introduce themselves by name.
- Thank the audience.
- State the title/topic of the presentation.
- Give the plan of the talk.
- Say how long the presentation will last.
- Give some background about the topic .
- Explain the purpose of the presentation.
- State the key findings.
- Discuss the applications of the research.

4. Correct the mistakes in the common phrases from a traditional presentation opening below. There is one mistake in each phrase.

1. Greetings, ladies and gentlemens.
2. The subject of my today's presentation is…
3. Next I will say you…
4. After, I will discuss…
5. Finally, I will explain you…
6. If you have any question, please ask at any time.

5. Can you think of any more interesting ways to start a presentation?

6. Plan the introduction to your next presentation. You should do some or all of the following:

- Introduce yourself by name
- Thank the audience
- State the title/topic of the presentation
- Give the plan of the talk
- Say how long the presentation will last
- Give some background to the topic
- Explain the purpose of the presentation
- Use attention grabbers

7. Practice delivering your introduction to the class.

10.4. Using slideware

8. Discuss the different software options for making slides for mathematical presentations. What are their advantages and disadvantages?

9. Discuss the slides (1–3) from mathematical presentations at the end of this unit. What do think is good about each slide? What could be improved?

10. Look at the full slide deck for Slide 2. How does the presenter solve the problems mentioned in the information box [21]?

When discussing slide presentations in mathematics, we usually make the following points:

- When slides contain large amounts of text (or equations), the audience cannot read and listen at the same time, so strategies are needed either to reduce the content on the slides or to guide the audience through the content.
- The audience needs time to absorb math concepts, but it is very easy to click through slides too quickly, especially when the presenter is nervous, so strategies are needed to give the audience time to think.
- The audience cannot refer to past slides to remind themselves of the meaning of new notation or of the purpose of details being presented, so strategies are needed to help the audience remember important points.

11. Prepare a slide deck for your next presentation. Show your slides to a partner or the class and discuss how they could be improved.

10.5. Signposting language

Signposting language is crucial in math presentations as it helps the audience understand the structure of the talk and follow the speaker's logic. It provides clear and concise cues to the listeners regarding the topics covered, the main points discussed, and the connections between them.

12. Look at the list of useful phrases opposite. Choose one or two phrases from each section that you could use in your next presentation.

10.6. Presenting conclusions

13. Discuss whether you agree with this advice for presenting conclusions.

- Signal that you have reached the end of the talk, using a phrase such as “to summarize” or “to conclude”.
- Write ‘Thank you’ or ‘Thank you for your attention’ on the final slide.
- Outline future work or open questions.
- Make a link back to the start of your presentation.
- Keep your conclusion short and to the point.

14. Do you have any other advice for concluding a mathematical presentation?

15. Prepare the conclusion to your next presentation. Practice delivering the conclusion to the class.

Introducing the first point
So let’s start, shall we?
So, let’s start with …
I’d like to begin by saying / describing / explaining …
Firstly, let’s consider…
To begin with, …
To start with, …
First of all, I’ll …

Transitioning from one section to the next
That's all I have to say about...
So that concludes the part about ...
We've looked at... Now we need to…
Now that we’ve found… we need to calculate…
This leads me to …
The next issue/topic/area I’d like to focus on is …
Now we'll move on to ...
Now, let’s examine…
We now need to discuss ...
Turning to ...
Moving on…

Clarifying and simplifying
Put simply ...
In other words...
To put it another way ...
To put it more simply ...
Let me rephrase that to make it quite clear.

Giving examples
A good example of this is ...
As an illustration, ...
To give you an example, ...
To illustrate this point ...

Explaining visuals
This graph/image/video/data demonstrates …
I’d like to illustrate this by showing you …
As you can see in this picture, …
This diagram shows …
If you look at this diagram, you can see …
What is interesting in this slide is …
I’d like to draw your attention to …

Concluding
Well, that brings us to the end of the final section. Now, I’d like to summarise by …
To summarise ...
To conclude ...
In conclusion ...
Let's summarise briefly what we've looked at.
If I can just sum up the main points.
So, I think I’ve covered the main points.
That brings me to the end of my talk.

Managing a Q&A

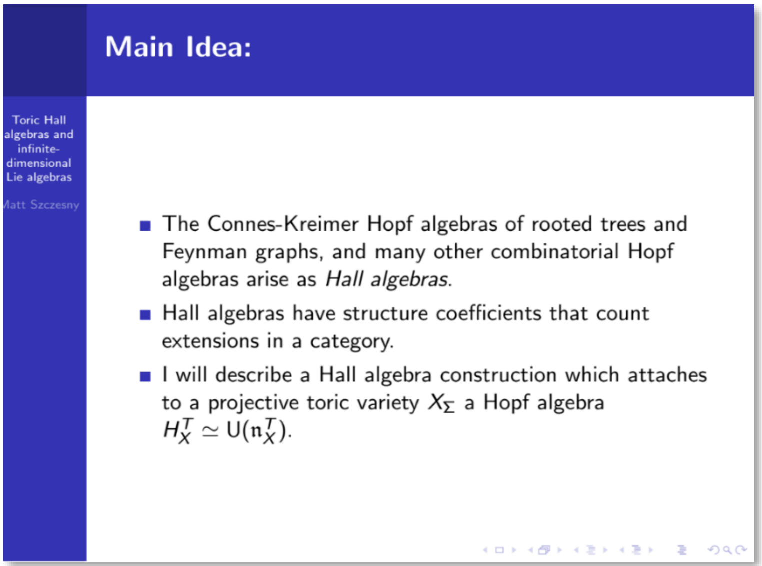
After many presentations, the audience has an opportunity to ask the presenter questions. Time for questions is often limited to about 10 minutes. Key skills include the ability to answer questions on the spot, to be concise, and to avoid debates with individual audience members.

16. What advice would you give to someone preparing for a Question and Answer (Q&A) session? Think about:

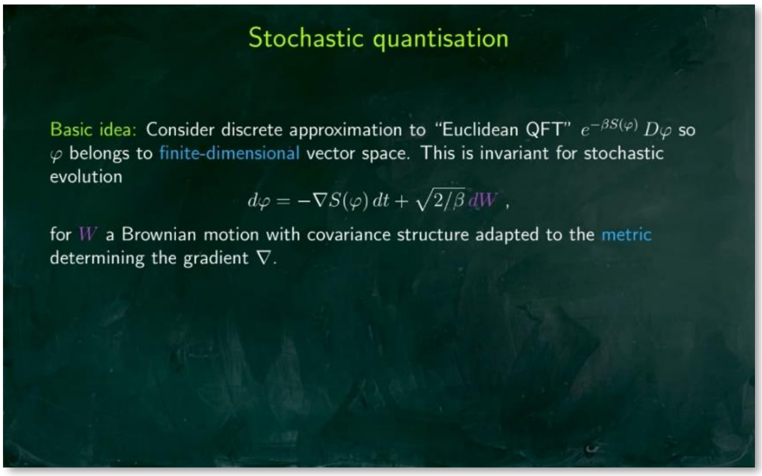
1. How to anticipate questions that might be asked.
2. How to guide your audience towards asking certain questions.
3. What to say if you do not understand the question.
4. What to say if the question is irrelevant, or the questioner has misunderstood the issue.
5. How to deal with a comment, not a question.
6. What to do if the question is too complex to give a short answer.
7. How to avoid being drawn into an argument.

17. Put the useful phrases for a Q&A session into the correct order.

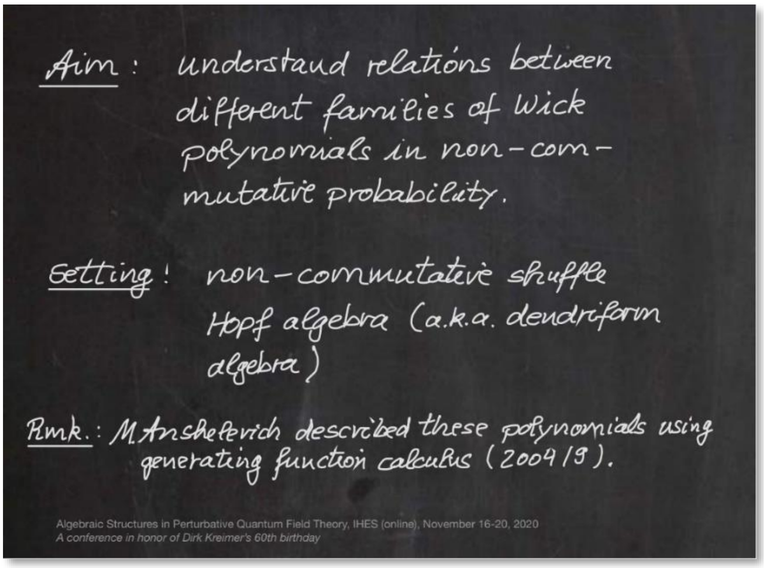
1. for a question, thank That's you asking it. great
2. So asking what me is… you're
3. For at back, the was… question those the
4. sorry, I'm not I sure understand, could repeat question? you the I'm
5. that your answer question? Does
6. need I about guess I to a bit that. more think
7. you for Thank your comment. anyone Does have any questions? more
8. I time I would think more to that answer question. need
9. break? Would like to meet to you discuss during the coffee that
10. you? Could email me you and I'll get that back to question



Slide 1. Toric Hall Algebras and infinite-dimensional Lie algebras [31].



Slide 2. Random Loops and T-algebras [11].



Slide 3. Wick Products and Combinatorial Hopf Algebras [8].

10.7. General feedback

Presenter: ______________________________ Date: __________________________

1. What did you particularly like about the presentation?

2. Which part of the presentation was the most difficult to follow?

3. What advice do you have for the presenter for next time?

4. What advice do you have for yourself? What do you want to remember as you prepare and/or deliver your next presentation?

10.8. Student quiz [2]

Each presenter comes up with 2–5 questions that anyone who attended the talk should be able to answer. The questions should get at the main point that the speaker hopes the audience will take away from the talk. They should be relatively easy and not involve too much notation.

During the presentation, the audience tries to answer the questions. At the end of the talk, they check their answers in groups.

Finally, the presenter decides if the answers are correct.

10.9. Three things [36]

"Three Things" is an exercise to learn how to get things out of talks. The theory is that if you can get even three things out of a talk, it is a success. And if you can't get even three small things out of a talk, it is not. The "things" can be of many forms:

- a definition you want to remember
- a theorem you want to remember
- a motivating or key example
- a motivating problem
- a question you want to ask the speaker
- a question you want to ask someone else

anything else of a similar flavor: something specific that made you think. Something vague ("I liked the part where she talked about groups") does not count as a "thing".

After the talk, post your things on a forum (or discuss them in person).

10.10. Questions and answers

Think of a question to ask after the next presentation you see. If you want to have fun, ask a question that is irrelevant, illogical, or complex, to see how the speaker responds.

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